RATE OF CHANGE

EXAMPLE: A company determines that the cost in dollars to manufacture x cases ofCD's "Imitations of the Rich and Famous" by Kevin Connors is given by $C(x) = 100 + 15x - x^2$, $0 \le x \le 7$

a) The **Change in Cost** as the production increases from 1 to 5 cases:

_________________________________ = ___

b) The **Average Change of Cost** as the production level increases from 1 to 5 cases:

_____________________________ = ________ = ______

Thus, on the average, the cost increases at the rate of ______ per case when theproduction level increases from 1 to 5 cases.

c) Find the average change in cost as the production goes from 5 to 7 cases.

_________________________________ = ________

Thus, on the average, the cost increases at the rate of ______ per case when the production level increases from 5 to 7 cases.

EXAMPLE: Find the average rate of change of $f(x) = 2x^2 - x - 3$ between the values of $x_1 = 2$ and $x_2 = 2.5$

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********AVERAGE RATE OF CHANGE** from x to x+h is ********

$$\frac{f(x+h) - f(x)}{(x+h) - x} = \frac{f(x+h) - f(x)}{h}$$

slope of sec*ant* $m_{\sec}$

This is called the **DIFFERENCE QUOTIENT** (or and average rate of change)

THE DERIVATIVE

For a function f(x), the **derivative**, $f'(x)$, can be found by this TWO-STEP definition method of the derivative.

STEP 1: From the difference quotient $\frac{f(x+h)-f(x)}{h}$

A) Find and simplify f(x+h)
B) Find and simplify f(x+h) - f(x)
C) Find and simplify $\frac{f(x+h)-f(x)}{h}$ (notice that when step 1C is completed, that a factor of *h* has been canceled from numerator and the denominator)

STEP 2: Evaluate $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ = slope of the tangent line = $m_{\tan}$

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example 1: use the two-step definition method to find the following:

a. $f'(x)$ b. $f'(2)$ c. $f'(-5)$

$f(x) = x^2 + 6x$

step 1A____

step 1B ____

step 1C ____

step 2 ____

a. ______ b. ________ c. _________

example 2: use the two-step definition method to find the following:

a. $f'(x)$ b. $f'(2)$ c. $f'(-5)$

$f(x) = 3x - x^2$

step 1A____

step 1B ______________________________________

step 1C ______________________________________

step 2 ____

a. ______ b. ________ c. _________

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NONEXISTENCE of the derivative:

The existence of a derivative at x = a depends on the existence of a limit at x = a, that is , on the existence of $f'(a) = \dfrac{f(a+h) - f(a)}{h}$

If the limit does not exist at x = a, we say that the function f is **nondifferentiable at x = a**, or $f'(a)$ **does not exist**.

***** *see graphs b, c, d, e on page 183 of your text*

Given the function $f(x) = 3x - x^2$

1 find $f'(x)$ 1) ________

2. Find the slope of the line tangent to f(x) at x = 0 2)___

3. Find the equation of the line tangent to f(x) at x = 0 3) ____________

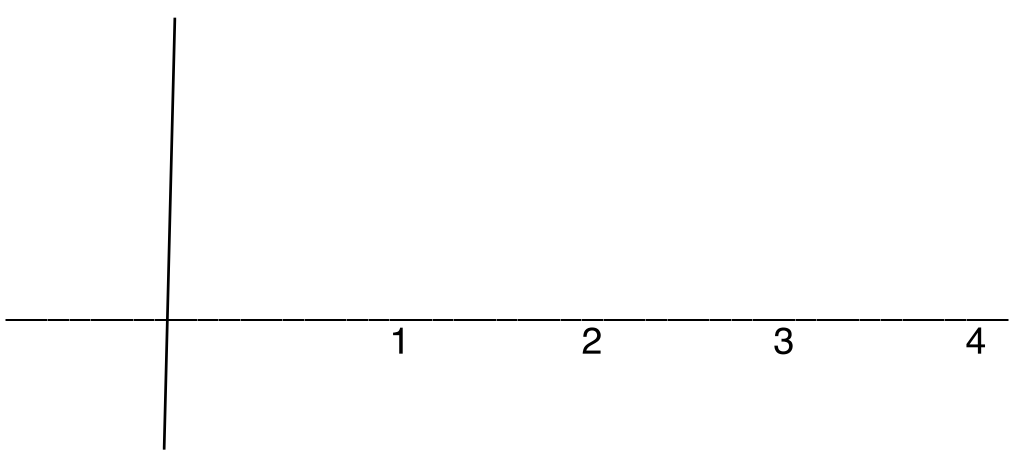
4. Find the slope of the line tangent to f(x) at x = 1 4)___

5. Find the equation of the line tangent to f(x) at x = 1 5) ____________

6. Find the slope of the line tangent to f(x) at x = 2 6) ____

7. Find the equation of the line tangent to f(x) at x = 2 7) ____________

8. Graph f(x) and sketch these tangent lines in parts 3, 5, and 7



Short cut rules to find the derivative

FUNCTION **DERIVATIVE**

f(x) = C (a constant) f'(x) = _________

f(x) = Cx+b f'(x) = _________

f(x) = x^r (POWER RULE) f'(x) = _________

f(x) = Cx^r f'(x) = _________

f(x)=$C \cdot f(x)$ f'(x)= ___________

f(x) = g(x) + h(x) (SUM RULE) f'(x) = _________

f(x) = g(x) - h(x) (DIFFERENCE RULE) f'(x) = __________

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(For chapter 3)

f(x) = $F(x) \cdot G(x)$ (PRODUCT RULE) f'(x) = _______________________

f(x)=$\frac{T(x)}{B(x)}$ (QUOTIENT RULE) f'(x) = ________________________

f(x) = $f(g(x))$ (GENERAL POWER RULE) f'(x) = ________________

(CHAIN RULE)

FUNCTION	**DERIVATIVE**
1. f(x) = -7	1.
2. y = π	2.
3. f(x) = -2x	3.
4. f(x) = $\frac{x}{2}$	4.
5. y = x	5.
6, y = x^5	*6.*
7. y = $\frac{1}{x^2}$	7.
8. f(x) = $6x^3$	8.
9. y = $\frac{3}{\sqrt{x}}$	9.
10. y =$5x^2 + \frac{1}{x}$	10.
11. f(x) = $\sqrt[3]{x} + \sqrt{x}$	11.
12. y = $6\sqrt{x} - 4$	12
13. y =$5x - 4x^{-2}$	13.
14. y = $\frac{1}{x} + \frac{1}{x^2}$	14.

EXAMPLES CONTINUED

16. Find y' for $y = \frac{3}{4x} + 2x^5 - 1$

17. Find $\frac{dy}{dx}$ for $y = 6\sqrt{x^3} + 7x$

18. Find $f'(x)$ for $f(x) = 8 - \frac{3}{\sqrt{x}} + \frac{2}{3}x^4$

19. Find $D_x\left(4x - x^2 + \frac{5}{x^3}\right)$

20. Differentiate the function $f(x) = \frac{4}{3x^2}$

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21. Given $f(x) = 2x^3 - 6x + 1$, find

a) $f'(x)$

b) the slope of the graph of f(x) at x = 2.

c) the equation of the tangent line at x = 2.

d) all value(s) of x where the tangent is horizontal (for any horizontal line, the slope is zero)

Examples of derivatives (continued)

22. The total cost of producing x dishwashers per week is given by

$C(x) = 4000 + 150x + \frac{x^2}{2}$ *dollars*

a) In economics, the word **margina**l refers to **rate of change**, that is, to a **derivative**. Find the **marginal cost** function for this example

b) Find the **marginal cost** at a production level of 10 dishwashers

c) Find the **actual cost** of producing the 11th dishwasher

d) Compare your answers in parts b and c above. Which is easier to calculate?

e) Calculate $C'(20)$ and interpret the result.

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23. In a psychological experiment, after t hours of training, a guinea pig learned N(t) basic skills where $N(t) = 5\sqrt{t} - \frac{1}{4}t$

What is the instantaneous rate of change of learning

a) after 4 hours of training?

b) after 9 hours of training?

MARGINAL ANALYSIS: approximating the change in the cost, revenue or profit, etc., that results from a 1-unit increase in production

Basic Concepts: x is the number of units produced in some given time interval

COST FUNCTION____

MARGINAL COST:_______________________________________

DEMAND FUNCTION___________________________________

REVENUE FUNCTION:__________________________________

MARGINAL REVENUE:_________________________________

PROFIT FUNCTION:______________________________

MARGINAL PROFIT:_____________________________

AVERAGE FUNCTIONS:

average cost function:_______________________

average revenue function:_____________________

average profit function:_______________________

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EX 1. A manufacturer of leather wallets has determined that the total cost of producing x wallets each week is given by $C(x) = 40 + 5x + \frac{x^2}{4}$

a) Find the exact cost of producing the 31st wallet

b) show how the marginal cost closely approximates this cost

c) Find the average cost function $\overline{C}(x)$ and $\overline{C}(30)$ interpret result

d) Find the marginal average cost at x = 30 and interpret

e) Find the average cost and marginal average cost at x = 40 and interpret.

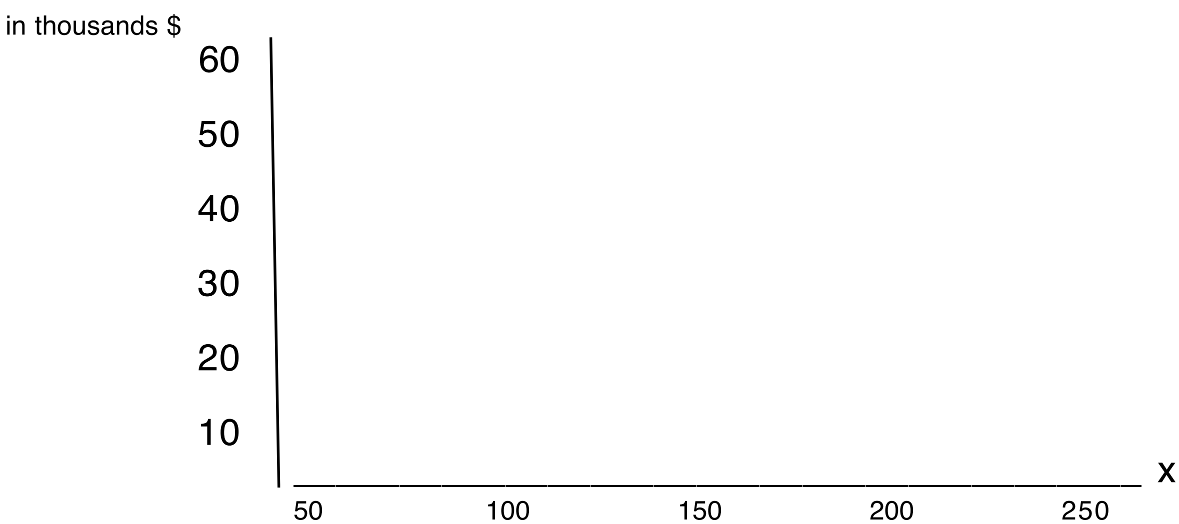
EX 2. The sales department of a camera manufacturing company has determined that if the cameras sell for $600 each, the company can sell 100 cameras each week. If the price is reduced to $400, then 150 cameras can be sold each week. the company has weekly fixed costs of $10,000 and each camera costs $200 to produce.

a) Assuming that the demand equation is linear and find it.

b) Find the revenue function

c) Assuming that the cost equation is linear, find it.

d) Graph the cost function and revenue functions on the same set of axes. Find the **Break-even** points and indicate the regions of profit and loss.



e) Find the profit function

f) Find the marginal profit at $x = 90$, $x = 100$, and $x = 110$. Interpret the results.

g) If you were the CEO of this company and you had the information from part f) above, how many cameras would you want to produce each week and why?