1. Find $f'(x)$ and simplify

a) $f(x) = (x^3 - 2x)(4x - 1)$

b) $f(x) = \dfrac{6x^2 - x + 3}{x^3 - 5}$

>>>

2. Find *f* ?(*x*) and do **not** simplify

a) $f(x) = \left(\sqrt{x} + 3\right)\left(\dfrac{4}{x} + 3x^2\right)$

b) $f(x) = \dfrac{8 - \sqrt[3]{x^2}}{\dfrac{5}{x} - 2x}$

>>>

3. Find $f'(x)$ and the equation of the tangent line at x = 3

a) $f(x) = (x + 2)(x^2 - 5)$

b. $f(x) = \dfrac{3x - 1}{4x + 7}$

DERIVATIVES OF PRODUCTS AND QUOTIENTS (continued)

4. Given $f(x) = \frac{x^2 + 25}{x^2}$

a) Simplify f(x) first, and then find the derivative.

b) Without simplifying f(x), use the quotient rule to find its derivative.

>>>

5. For the function $f(x) = \frac{x^2 + 1}{x^2 - 1}$

a) Find $f'(x)$ and simplify the numerator

b) Find the value(s) for which $f'(x) = 0$

6. For the function $f(x) = (x + 7)(5 - x^2)$

a) Find $f'(x)$ and simplify

b) Find the value(s) for which $f'(x) = 0$

>>

7. An employee can accomplish N(t) tasks after t hours of training where $N(t) = \frac{100t + 200}{t + 32}$

a) find $N'(t)$, the **instantaneous rate** of work output

b) find $N'(18)$ and interpret the results

c) find $N'(68)$ and interpret the results

A RULE FOR DIFFERENTIATING POWERS OF FUNCTIONS

If f(x) is a composite function such as $f(x) = \left(function\ f\ \right)^n$ where *n* is a constant, then $f'(x) = n\left(function\ f\ \right)^{n-1}\left(derivative\ of\ f\ \right)$

ie: If $y = f(g(x))$ then $f'(x) = f'(g(x)) \cdot g'(x)$ if they; exist

>>

example 1 : $y = \left(x^3 + 5\right)^2$ then $y' = 2\left(x^3 + 5\right)^1\left(3x^2\right)$

question: Where did the factor of ($3x^2$ *) come from?*

answer: Look at $y = \left(x^3 + 5\right)^2 = \left(x^3 + 5\right)\left(x^3 + 5\right)$ and use the Product Rule to take the derivative. The result is $y' = \left(x^3 + 5\right)\left(3x^2\right) + \left(x^3 + 5\right)\left(3x^2\right) = 2\left(x^3 + 5\right)\left(3x^2\right)$

>>>

example 2: $y = \left(x^3 + 5\right)^{10}$ The derivative is $y' = \_\_\_\_\_\left(x^3 + 5\right)^{?}(\_\_\_\_)$

>>

Find the derivative of each

3. $h(x) = \left(x^3 + 5\right)^{\frac{1}{3}}$

4. $h(x) = \dfrac{1}{\left(x^3 + 5\right)^9}$

5. $h(x) = \left(x^2 - 7x + 1\right)^5$

6. $h(x) = \dfrac{2}{\left(x^3 - x\right)^4}$

7. $g(x) = \sqrt{5x - 2}$

8. $f(x) = \dfrac{3}{\left(4x^2 + 6x - 5\right)}$

COMBINING RULES:
1. find the derivative of each

a. $f(x) = x^3\left(x^4+3\right)^2$

b. $f(x) = \dfrac{5x^2}{(6x-7)^3}$

c. $g(x) = 2x^2\sqrt{8-x^3}$

2. Find an equation of the tangent line to the graph of $y = x\sqrt{1+x^3}$ at $x = 2$

>>
APPLICATIONS:
1. A manufacturer's total cost in producing x items is
(in thousands of dollars)

a) Find the rate of change of the total cost function

b) Find $C'(3)$ and interpret

c) Find $C'(5)$ and interpret

2. A small polluted lake was treated with a chemical to destroy bacteria. It is estimated thatafter *t* days of treatment, the number of bacteria in one CC should be *N(t)*, where $N(t) = 500(8-t)^2$

a) Find $N'(t)$

b) Find $N'(1)$ and $N'(6)$ and interpret each.

3. It is estimated that t years from now, the population of a certain town will be $p(t) = 20 - \frac{6}{t+1}$ thousands

A study indicates that the average daily level of carbon monoxide in the air will be $c(p) = 0.5\sqrt{p^2 + p + 58}$ ppm when the population is *p* thousand

a. At what rate will the carbon monoxide level be changing with respect to population when the population is 18 thousand people?

b. At what rate will the carbon monoxide level be changing with respect to time 2 years from now?

MATH 105

LOCAL EXTREMA AND THE FIRST DERIVATIVE TEST

Finding local (or relative) extrema for a nonconstant function continuous on the interval (a,b)

1. Find the domain of the function
2. Find the **Critical Values**, *c*, of the function: values of x <u>in the domain</u> of the function that make $f'(c) = 0$ or $f'(c) = ND$
3. Use a <u>sign analysis</u> for f' and label the Critical Values.
   The critical values divide the f' x-number line into intervals.
   Test a value from each interval into f' to determine if f' is positive or negative throughout that interval.

   If the sign of f' changes
   from + to -, then f(c) is a local maximum
   from - to + , then f(c) is a local minimum
   no change at x=c means that there is not a local extremum at c

>>

Use the First Derivative test to determine where f(x) is increasing , decreasing , and local extrema

1. $f(x) = x^2(x-1)^3$

2. $y = -x^3 - 8x^2 + 9x - 6$

First Derivative test (continued)

3. $f(x) = (x+4)^{\frac{2}{3}}$

4. $f(x) = \dfrac{x-5}{x+3}$

>>>
A manufacturer of sofas finds that the total cost of producing x sofas per week is given by

$C(x) = 20 + 200x + \dfrac{x^3}{100}$, $0 < x < 25$.

a. Find the Average Cost function

b. Find the intervals where the Average Cost is decreasing and increasing and all local extrema

Math 105 Section 3.5 Absolute Extrema

Example Find the absolue extrema for $f(x) = 2x^3 - 3x^2 - 12x + 24$

a. for [-3,4]
b. for [-2,3]
c. for [-2,1]

Example: A company manufactures and sell x radios per week. The revenue received for selling x radios per week is modeled by $R(x) = 10x - 0.001x^2$ for $0 \le x \le 10{,}000$.

a. find the production level that will yield the absolute maximum revenue

b. Give the absolute maximum revenue

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