The science of uncertainty is called **probability theory**. The probability of an event is its long-run relative frequency. An **experiment** is an action whose outcome cannot be predicted with certainty.

Experiment: From a bag of 5 marbles, 1 red, 1 green, 1 blue, 1 yellow, 1 pink, draw 2 marbles, **without** replacement.

A **sample space** is the set of all possible outcomes of an experiment. The sample space for this example is

How many different outcomes are there? ____

An **event** is an outcome or group of outcomes. If event E is having one red marble in the draw then event E consists of the outcomes ____________.

Each outcome of this experiment is **equally likely** to occur. Therefore each outcome has the same probability or likelihood of occurring.

The probability of drawing one green and one red is ____.

When outcomes are equally likely, the probabilities are **relative frequencies** or percentages. Then the probability of an event is the ratio of the number of ways the event can occur, $f$, to the number of possible outcomes, $N$.

The probability of the event of having exactly one red marble in the draw is

$$
\frac{f}{N} = 
$$

The **Law of large Numbers** is the **frequentist interpretation** of probability. It is the interpretation that the probability of an event to be the proportion of times the event occurs in a large number of repetitions of the experiment.

**BASIC PROPERTIES OF PROBABILITY:**

1. The probability of an event is always between 0 and 1.
2. The probability of an event that cannot occur, an impossible event, is 0.
3. The probability of an event that must occur is, a certain event, is 1.
Sample spaces, events, and relationships between events are often pictured by means of **Venn Diagrams**. The sample space is represented by a rectangle, events are represented by circles.

The probability of a simple event, $A$ is denoted by the circle below. All of the outcomes in event $E$ are within the circle.

The rectangle represents the sample space. (probability=1)
The circle represents the event $A$.
The shaded region represents the **complement** of event $A$, denoted $A^C$.

$$P(A) + P(A^C) = 1$$

**Disjoint** events are events with no outcomes in common. Two disjoint events, $A$ and $B$, are pictured as the two circles below.

These disjoint events are **mutually exclusive**, meaning they cannot occur at the same time.

$$P(A \text{ and } B) = 0$$

**EX.** Roll a die. Let event $A$ be the event of rolling a 3. Let event $B$ be the event of getting an even number. $A = \{3\}$ $B = \{2, 4, 6\}$

(Where $P(A) = \frac{1}{6}$ and $P(B) = \frac{3}{6}$)

Events $A$ and $B$ are disjoint events. They are mutually exclusive.

When a die is rolled, the probability of event $A$ and event $B$ occurring at the same time is $P(A \text{ and } B) = ____$.

For mutually exclusive events, the probability of event $A$ or event $B$ occurring (in this example, that is the event of rolling a 2, 3, 4, or 6) can be written using the **Special Addition Rule**:

$$P(A \text{ or } B) = P(A) + P(B)$$

(in this ex: $P(A) + P(B) = \frac{1}{6} + \frac{3}{6} = \frac{4}{6}$)

**Probability and the Multiplication Rule**

Two events $A$ and $B$ are **independent** if the occurrence of one does not affect the probability of the occurrence of the other. If the outcome of $A$ (or $B$) does affect the outcome of the other, then $A$ and $B$ are **dependent** events. (Several events are also similarly independent if the the occurrence of one does not affect the probabilities of the occurrences of the other. Several events are also similarly dependent if the occurrence does affect the occurrence of the other)

**example:** tossing a die and flipping a coin are independent events because the outcome of the die has no effect on the outcome of the coin.

**example:** the event of having your car start and the event of driving to work on time are dependent because the outcome of trying to start your car does affect the probability of getting to work on time.

**Special Multiplication Rule:** $P(A \text{ and } B)$

Are $A$ and $B$ independent? **YES**...then....

$$P(A \text{ and } B) = P(A) \cdot P(B)$$
EXAMPLES
1. Sue is an actuary working for the Life Trust Insurance Company. She needs to find the probability of a 20-year-old male living to be 30 years of age. She surveys 1000 20-year-old males and then counts those who live to be 30. She finds that 984 of them lived to be 30. She then uses a relative frequency approach to probability to estimate this probability for all 20-year-old males. Find this probability.

2. A consumer test group consists of 80 persons, 30 of whom are women. If we randomly select one person from this group, find the probability that the person is not a female.

3. If a year is selected at random, what is the probability that Thanksgiving Day will be on
   a) a Wednesday? _____
   b) a Thursday? _____

4. Survey subjects used in market research are often chosen by using computers to randomly select telephone numbers. Assume that a computer randomly generates the last digit of a telephone number, find the probability that the outcome is an 8 or a 9.
   \[ P(8 \text{ or } 9) = \]
   We used what rule here? __________________ because the event of getting an 8 and the event of getting a 9 are __________________ events.

Chapter 15
General Addition Rule
If two events, A and B are not mutually exclusive, then they share outcomes and the circles in the Venn Diagram overlap and the Special Addition Rule cannot be used.

The shaded region to the left represents \( P(A \text{ and } B) \)

The General Addition Rule: \( P(A) \text{ or } P(B) = P(A) + P(B) - P(A \text{ and } B) \)

5. If \( P(A \text{ or } B) = \frac{1}{3} \), \( P(B) = \frac{1}{4} \), and \( P(A \text{ and } B) = \frac{1}{5} \), find \( P(A) \).

6. Find \( P(B) \) if \( P(A \text{ or } B) = 0.6 \), \( P(A) = 0.2 \) and events A and B are mutually exclusive.

7. If events A and B are mutually exclusive, and events B and C are mutually exclusive, must events A and C be mutually exclusive? Use a Venn Diagram to support your answer.
**Multiplication Rule** (con’t)

Independent events are those for which the occurrence of one event does not affect the probability of the occurrence of the other. (Two or more events cannot be both mutually exclusive and independent.)

**EXAMPLE 1:** Fifty hair dryers are produced. Forty of them are good and ten of them are defective. If we randomly select two of these dryers, what is the probability that both of the dryers are good. ie: find \( P(G_1 \text{ and } G_2) \)

CASE 1
with replacement (thus independence)
\[
P(G_1 \text{ and } G_2) = \frac{40}{50} \times \frac{39}{49}
\]

CASE 2
without replacement (no independence)
\[
P(G_1 \text{ and } G_2) = \frac{40}{50} \times \frac{39}{49} - \frac{40}{50} \times \frac{10}{49}
\]

**General Multiplication Rule**

Are \( A \) and \( B \) independent? **NO**...then...

\[
P(A \text{ and } B) = P(A) \times P(B | A) \quad \text{or} \quad P(A \text{ and } B) = P(B) \times P(A | B)
\]

**MULTIPLICATION RULES:**

If \( A \) and \( B \) are independent, then \( P(A \text{ and } B) = P(A) \times P(B) \)

If \( A \) and \( B \) are **not** independent then \( P(A \text{ and } B) = P(A) \times P(B | A) \) or
\[
P(A \text{ and } B) = P(B) \times P(A | B)
\]

**VENN DIAGRAM example:** page 356, ex 4
EXAMPLES:
1. Ninety percent of all motorcycle drivers are male. If a policeman stops 5 different motorcycle drivers at a license checkpoint, what is the probability that
   a) all are males?
   b) there is at least one female.

2. Five hair dryers are produced: four of them are good and one is defective.
   a) If we randomly select one of them, what is the probability that it is good?
   b) Suppose that two dryers are randomly selected for testing (and the first is replaced before the second selection is made). Find the probability that both selected dryers are good.
   c) Same as part b) but we do not replace the first selection before selecting the second dryer.

3. Let $P(A) = 0.75$ and $P(B) = 0.83$. Find $P(A$ and $B$) given that
   a) $A$ and $B$ are independent
   b) $P(B \mid A) = 0.50$
   c) $P(A \mid B) = 0.33$
   d) $P(A \mid B) = P(A)$

4: Approximately 9% of men have a type of color blindness that prevents them from distinguishing between red and green. If four men are selected at random, what is the probability that all four of them will have this type of red-green color blindness?

5: There are 6 defective fuses in a bin of 80 fuses. The entire bin is approved for shipping if no defects show up when 3 randomly selected fuses are tested.
   a) Find the probability of approval if the selected fuses are replaced before the next draw.
   b) Find the probability of approval if the selected fuses are not replaced before the next draw.

6 A Harris poll found that 46% of Americans say they suffer great stress at least once a week. If three people are randomly selected, what is the probability that all three will say that they suffer great stress at least once a week?
Bivariate data looks at the relationship between two variables. A Contingency table, or two-way table, is a frequency distribution for bivariate data.

Example: Suppose that 500 workers were sampled and asked for their opinion of the most serious economic problem in the United States. For each worker, his resident region was recorded along with his response.

We have two types of variables: geographic region and opinion. The results can be summarized as below:

<table>
<thead>
<tr>
<th></th>
<th>NE</th>
<th>MW</th>
<th>SW</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>REGION</strong></td>
<td>NE</td>
<td>MW</td>
<td>SW</td>
</tr>
<tr>
<td>Ethanol</td>
<td>57</td>
<td>53</td>
<td>44</td>
</tr>
<tr>
<td><strong>OPINION</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equal Employment</td>
<td>72</td>
<td>40</td>
<td>48</td>
</tr>
<tr>
<td>Trade Policies</td>
<td>71</td>
<td>57</td>
<td>58</td>
</tr>
<tr>
<td>Total</td>
<td>180</td>
<td>150</td>
<td>150</td>
</tr>
</tbody>
</table>

The values are the frequencies of the sample results. Each rectangle is called a cell. The numbers inside the cells are the observed cell frequencies.

1. Give the horizontal and vertical totals for the cells.

MARGINAL PROBABILITIES RELATE THE COLUMN AND ROW TOTALS TO THE TOTAL NUMBER OF THOSE SAMPLED.

2. What percentage of the sampled workers were from the northeast? _____

3. If a sampled worker is chosen at random, then what is the probability that he believes that inflation is the main economic problem? _____

JOINT PROBABILITIES RELATE THE CELL FREQUENCIES TO THE TOTAL NUMBER SAMPLED.

4. What percent of the sampled workers are from the Southwest and believe that the trade policy is the main problem with the economy? _____

5. If a sampled worker is chosen at random, what is the probability that he is from the Midwest and believes that unemployment is the main problem? _____
The probability that an event B occurs given that event A has occurred is called a **conditional probability**. It is denoted by the symbol $P(B \mid A)$ which is read “the probability of $B$, given $A$”

Example 1: Suppose that a survey is taken on campus about the smoking policy and we have:

- A is the event that a respondent is male.
- B is the event that a respondent favors a ban on campus smoking.

Then $P(B \mid A)$ can be read:

- “Given that a respondent is male, find the probability that he favors a ban on campus smoking.”
- “On the condition that a respondent is male, find the probability that he favors a ban on campus smoking.”
- “What is the probability that a male respondent favors a ban on campus smoking.”

Example 2: Calving records from one farm for the years 1997 - 2000 yield the number of males and females delivered at various times.

<table>
<thead>
<tr>
<th></th>
<th>Day</th>
<th>Evening</th>
<th>Night</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bulls</td>
<td>29</td>
<td>5</td>
<td>17</td>
</tr>
<tr>
<td>Heifers</td>
<td>17</td>
<td>9</td>
<td>16</td>
</tr>
</tbody>
</table>

Using **D,E,N,B,H**, write an expression for the probability of the following, then find the probability value.

What is the probability that:

1. a calf is a heifer?
2. a calf is either a heifer or is delivered during the day?
3. a calf delivered is a bull and is born during the evening?
4. a calf is delivered during the night?
5. a calf is a bull, given that it is delivered during the day?
6. a bull is delivered during the day?
7. a day time delivery is a bull?
### UNDERGRADUATE COLLEGE ENROLLMENT (thousands of students)

(Enrollment in U.S. colleges and universities in the fall of 1995)

<table>
<thead>
<tr>
<th>Age</th>
<th>2-year full-time colleges</th>
<th>2-year part-time colleges</th>
<th>4-year full-time colleges</th>
<th>4-year part-time colleges</th>
</tr>
</thead>
<tbody>
<tr>
<td>Under 18</td>
<td>41</td>
<td>125</td>
<td>75</td>
<td>45</td>
</tr>
<tr>
<td>18 to 24</td>
<td>1378</td>
<td>1198</td>
<td>4607</td>
<td>588</td>
</tr>
<tr>
<td>25 to 39</td>
<td>428</td>
<td>1427</td>
<td>1212</td>
<td>1321</td>
</tr>
<tr>
<td>40 and up</td>
<td>119</td>
<td>723</td>
<td>225</td>
<td>605</td>
</tr>
<tr>
<td>Totals</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. How many undergraduates enrolled in colleges and universities? _____

2. What percent of all undergraduate students were 18 to 24 years old in the fall of 1995? _____

3. Find the percent of the undergraduates enrolled in the 4-year full time program. _____

4. An association of two-year colleges asks “What percent of students of two-year part-time colleges are 25 to 39 years old?” _____

5. A bank that makes education loans to adults asks: “What percent of all 25 to 39-year old students are enrolled part-time at 2 year colleges?” _____

6. The traditional college age group is ages 18 to 24 years. What percent of all undergraduates fall in this age group? _____

7. What percent of all part-time students are considered traditional students? _____